Teacher Knowledge for Maths Aotearoa Book 1B and Wilkie Way

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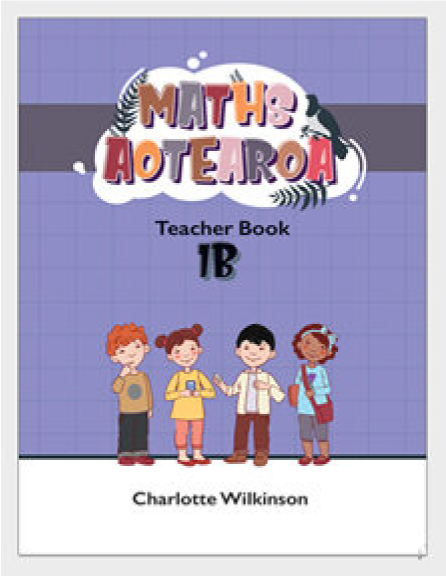

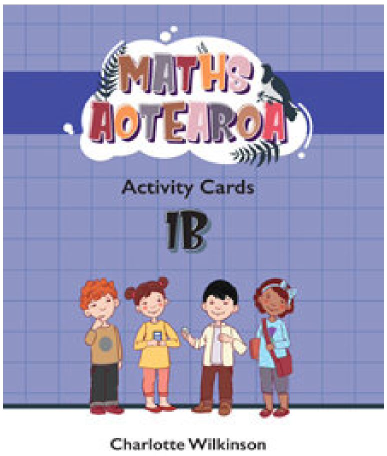

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Most Wilkie Way resources are available to download from wilkieway.co.nz through an annual subscription. Individual membership allows for personal use within your own class only.
School membership allows all teachers in a school to create an account and resources can be used across the whole school.
Teacher Handbooks for Fractions, Decimals & Percentages, Numbers & The Number System and Arithmetic Operations, ready to use laminated card Dice & Counter games and Assessment Screen booklets are available from the onlne store at wilkieway.co.nz

Maths Aotearoa Book 1B is organised into four large sequenced units.

Unit 1: Understanding Addition & Subtraction
Unit 2 Larger Numbers & Beginning Multiplication
Unit 3 Beginning Fractions
Unit 4 Beginning Place Value - Unlocking the Number System

Each unit within Maths Aotearoa Book 1B is made up of 6 elements. The best practice for delivery of content should be through a mixture of explicit teaching, guided practice, flexible grouping and independent activity, including play based activities with interactive dialogue to develop deeper thinking and language development in meaningful contexts. The elements within a unit are interconnected so it is important that connections are made to develop a robust foundational understanding of mathematics not just number, or space or measurement or statistics.

These units follow on from the units in Book 1A

Each unit has suggested classroom activities, many using equipment readily available in a junior classroom. There are also a total of 100 activity cards to support units 1 - 4. Many of the activities are practical and can be used multiple times. Some have activities on both sides of the cards and some have teacher information for further teacher guidance in developing the mathematics from the activity.

Further practice for each unit can be found in 13 printable number and algebra workbooks available to Wilkie Way members.

Tracking Progress:

Maths Aotearoa provides tracking sheets to record student progress through out their learning journey.

Progress tracking sheets can also be found under the Assessment heading in the Wilkie Way membership directory. These tracking sheets have been updated to reflect the refreshed curriculum by using the 6 headings of the refreshed curriculum rather than the three headings of the previous curriculum. The mathematics hasn't changed!

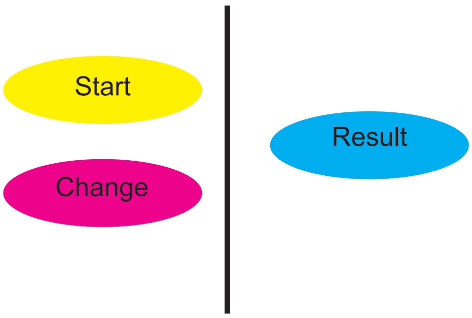
Maths Aotearoa Book 1B

Unit 1 - Understanding Addition and Subtraction:

Adding & Taking Away (Structure 1)	Patterns & Relationships (Structure 2)	Complementary Addition & Difference (Structure 3)
Focus on Making 10	Exploring Statistics & Probability Continued through Unit 2	Exploring Position & Orientation Continued through Unit 2

To understand addition and subtraction, students need to have attained the concept known as the cardinal principle. This means they recognise that the last number in the counting sequence tells them how many objects are in the group. Once students achieve this concept they no longer demonstrate a need to count from one when faced with joining two sets together. They can begin a count from any number to find how many altogether.
Addition and subtraction make sense and student will begin to memorise specific facts. It is likely they have already memorised some doubles facts (as met in Maths Aotearoa Level 1a)
This unit focuses on developing a conceptual understanding of addition and subtraction and using the symbols to represent addition and subtraction situation.
There are three different structures for addition and subtraction situations. All structures have three numbers, anyone of which can be the unknown.

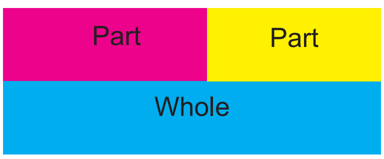
Structure 1



Join: Start amount, Change - the bit being added, Result – the total amount (because addition is commutative the start and the change can be swapped around without affecting the result – it is only for efficiency that we start with the larger number.)

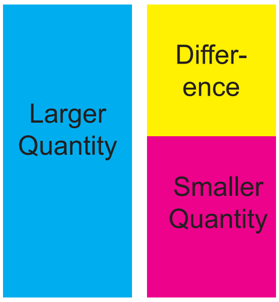
Separate: Start is the largest amount, Change – the bit taken away, Result – the amount left. (Subtraction is not commutative)

Structure 2



This structure leads to the idea that numbers are embedded in other numbers. It is the structure required to support the concept of a family of facts and subtraction being the inverse of addition.
Using number strips or Numicon are an ideal resources for this structure.

Structure 3



This third structure involves the comparison of two quantities. The third quantity does not actually exist. It is the difference between the two amounts.
Additive comparison problems are the most difficult for students to comprehend, possibly because of the non-existence of the third quantity.

All three structures can be represented in word problems and all three structures should be connected together when using physical materials like Numicon, number strips, tens frames etc.
Using only counters tends to lead to the over emphasis of the first (and most simple structure) being addition and subtraction and particularly for subtraction is likely to keep students counting from one as they count out a set of objects, count the ones taken away and then count the ones left.
Students should be encouraged to notice connections between addition and subtraction from the outset.

Mathematics is becoming more representative so a focus must be given to representing mathematical situations through words, pictures and numbers and simple equations. Students are beginning to make sense of symbols as representing mathematical situations.
Being able to draw a picture or model a situation with materials tells you students are able to visualise the problems and they can go on and solve the addition or subtraction by counting.

Using an equation to represent a situation is making more abstract demands. The symbols represent concepts and processes that have been built up through an appropriate range of experiences and dialogue about what is occurring. Mathematical meaning must be acquired before any form of symbolism is introduced.

Early experiences of mathematical symbols, used as a way of presenting work to students with limited reading ability will lead students into learning the symbols as an instruction to do something rather than a way of communicating numeric relationships.
The addition and subtraction symbol are operational symbols. They tell you to perform a specific operation. Most students presented with an equation will use the first addition/subtraction structure to give an answer. The equals sign is then learnt as an operation symbol meaning give the answer rather than as a relationship symbol representing equality of either side of the equation.

Overuse of completion of equations is likely to result in unintended learning consequences. To learn the meaning of the symbols, students need plenty of practice in writing the equations for specific situations most often given in simple word problems.
The problems should be written using familiar language and simple syntax as students need to take responsibility for reading mathematical problems from the outset. As students develop their reading skills, use mathematics as a genre for reading. It requires a high level of comprehension. Provide and assist students to read more complex word problems requiring the use addition and subtraction in all the different structures.
Give students the opportunity to create their own word problems. Students are likely to require assistance in formulating the language to describe the addition or subtraction scenario. As they become more proficient with the language to describe a situation, they begin to reflect a growing understanding of the concepts of addition and subtraction. Evidence of understanding should be reflected in the increasing variety and complexity of their addition and subtraction number stories.

Graphing, pictographs and bar graphs provide a good context for structure three, finding the difference by answering questions like: How many more? How many less? This same structure will be met again in Unit Three where measurement is the context when answering questions like: How much longer? How much shorter?

Unit 2 Larger Numbers & Beginning Multiplication

Sequencing to 100	Number Patterns with Multiples	Equal Grouping & Repeated Addition
Doubles & Halves	Exploring Statistics & Probability Continued from Unit 1	Exploring Position & Orientation Continued from Unit 1

Foundations for multiplicative thinking begin early in a student's learning and are found in counting (focus of this unit) and measuring (focus of unit 3) experiences. In unit 4 of level 1a students were exposed to equal grouping and some of the activities in this unit are a repeat of activities suggested in 1a unit 4.

An understanding of quantity provides the foundations for understanding multiplication. A quantity is a characteristic of objects that can be counted or measured. The number word describes the number portion of quantity. A number on its own does not exist. It can only be described in relation to something – the unit, for example counters. A count is a discrete quantity that answers the question How many? Counting has already begun with counting in ones and now progresses to counting in equal size groups. The unit component of a quantity is essential for solving multiplication type problems. Additive situations most often involve joining two quantities of the same unit (3 lollies and 5 lollies).

In a multiplicative situation the two quantities often refer to different units. (e.g 3 lollies in each of 5 bags). In visualising the problem the correct quantity unit must be attributed to each number. Understanding the quantity unit can be one of the comprehension barriers when reading multiplication type problems, which is why it is important to use meaningful contexts for problems.

Equal grouping problems can be viewed additively as equal additions with the student focusing only on adding the quantities of the same unit. It is important to represent the problems with materials or in pictures to ensure students focus on the quantity unit and not just the number. Students need to learn to co-ordinate the three aspects of a multiplicative situation: Group size, the number of groups and the total amount. Introduce students to a multiplicative representation of equal groups called an array.

Compared to the time taken to develop additive thinking, the time taken to develop multiplicative thinking may take many years. Students normally develop the ability to add and subtract naturally but multiplication is much more complex. This early stage focuses mainly on equal grouping but using vocabulary like twice, three times in every day situation pushes the idea of multiplicative comparison thinking.

This unit explores the idea of counting in equal groups as the beginnings of solving multiplicative type problems. Students can learn to skip count by rote without any understanding that the counting set is an equal group of more than one so being able to recite a skip counting sequence is not evidence of an understanding of equal grouping.
Many students find counting in multiples of two relatively easy, especially up to 10 or 12 as counting sets of objects in this range is a common everyday occurrence. The teen numbers can cause a problem when counting in ones so it is understandable that they can also cause a problem when counting in multiples of two.
At this stage in a student's learning they are more likely to see learning doubles as an addition idea but the word double is a multiplicative word. Unless this word is fully explored with repeated doubling students will thinking doubling is the same as repeated addition. (Repeated halving occurs in unit three when exploring fractions)

The idea of an equal group is used again on a measurement scale (the repeat of identical units). At higher levels these units of repeat will involve multiples (in the decimal system multiples of 10). A scale is also used on the axis of graphs. At higher levels these are likely to involve multiples.

The idea of an equal group of 10 is fundamental to the place value system of numbers. Students need to be able to manipulate the number of groups and the group size of ten. While one group of ten gives the same answer as ten groups of 1, students must maintain the group of ten to be able to use ten as a counting set. (This specific grouping will be revisited in unit four)

Sequencing numbers to one hundred in English requires the learning of 29 different words. The similarity in sound between for example thirteen and thirty leaves many students with confusion. Students' need plenty of practice in rote counting but they also need to be able to image the sequence – having a 0 – 100 number line on display is really helpful. The 100 chart while taking up less space does not provide the visual sequence of how far or near a number is to another number in the sequence.

In order to avoid or rectify a teen/ty confusion, students also need to consider the cardinal aspect of the numbers meaning the size of the number. Number 13 represents 10 + 3 (Additive structure of place value) and 30 represents 3 groups of 10 (multiplicative structure of place value). Students at this point have not considered 10 as representing one group of 10. Introduce the idea and it will be followed up further in unit four.
Te reo Maori supports both the additive structure of place vlaue 10 + 3 tekau ma toru and the multiplicative structure 3 groups of 10 toru tekau

The hundreds chart can begin to show each row as a group of ten and the decade numbers counting the groups of ten. The student must now begin the notice the position of a digit in a number and know whether they are working with a group of one or a group of ten.
A students understanding of a two digit number must expand :
For example: Students need to understand 24 as:
24 is 24 ones (cardinal)
24 is the number after 23, before 25 and between 23 and 25 (ordinal)
24 is 2 tens and 4 ones (Column position - linguisitc place value)
24 is 2 groups of ten and 4 groups of 1 (conceptual place value)

Further work on place value specifically is covered in unit four.

Unit 3: Beginning Fractions

Fractions of Shapes	Equal Sharing	Fractions of Numbers
Reflective Symmetry	Exploring Measurement Continued through Unit 4	Exploring 2D & 3D Shapes Continued through Unit 4

The gradual building of fractional knowledge begins with understanding fractions as partitions or divided quantities.
To develop a conceptual understanding of fractions, students need to revise their understanding of partitioning. Students have met partitioning in the context of addition and subtraction where the size of the parts does not matter. Partitioning into equal sized parts is the fundamental concept for understanding fractions, percentages and decimals. The key idea behind partitioning involves dividing a number or objects into equal sized groups (a discrete context) or partitioning an object or shape into same size pieces, (a continuous context). It is important to use a variety of representations for modelling partitioning to ensure students are thinking about the key ideas being taught and not simply memorising images or procedures to solve problems.

Equal sharing is the beginnings of partitioning a quantity or objects (discrete context for fractions)

Students need to begin to move from the counting strategy, (as met in level 1a) where they count out the total number of objects into the required number of groups and then count the number in each group. Thinking must move to foundational multiplicative thinking. Additive thinking does not support equal sharing and fractional thinking. Additive thinking is likely to lead to misconceptions as students continue to apply whole number thinking to fractions.

Repeated subtraction can be used for equal grouping situations where the size of the equal group is known, but in equal sharing the size of the group is unknown; the known is the total and the number to be shared between. While repeated subtraction would give a correct answer is does not model the situation of equal sharing. Equal sharing is mathematically more complex than equal grouping as it requires the foundational multiplicative thinking.
The student needs to consider of the total number of items to be shared, the number to be shared between and the number of items in the equal group received. Like for multiplication, students are being asked to manipulate all three numbers at the same time. Using an array structure (as introduced in unit two) promotes the use of multiplicative thinking by encouraging students to visualise the equal share and the equal groups that make up a set of objects.

Eventually students recognise the multiplicative relationship between the total, the number of groups and the number in each group and are able to use the inverse of multiplication, called division. (For most students this will take many years and lots of multiplicative experiences.)

Representations for the continuous context of fractions come from working with partitioning shapes. Using different shapes to show how some shapes can be halved in more than one way. Some can be halved again to make quarters some cannot. It is just as important to have experiences of not halves and not quarters for students to generalise the need for each part to be equal.

Halving shapes leads to work on exploring reflective symmetry. A repeated halving leads to quarters and two lines of reflective symmetry.

Activities for measuring length provide a visual context for continuous fractions. A measure (e.g. Length) is a particular type of quantity that has a continuous characteristic. Students must come to the understanding each unit of length used to measure a length must be an equal length repeated over and over again, without gap or overlap. With increasing number knowledge and experience of measurement by direct comparison students come to the understanding that another object can be used repeatedly and the number of times it is used can be counted to give a measurement. The essential concept is the object used must be of an equal measure (e.g. length) and the number of objects used must be counted. Measure also requires the understanding that a unit can be broken into a number of smaller equal sized units. At this level smaller units would involve halves and quarters of the unit of measure.
While measurement concepts are the same for all measures, it is the measurement of length that provides the necessary visual context for the concepts. Measuring mass, capacity, angles, temperature and time all require a considerable amount of language which continues to be a focus of measurement experiences.

Understanding measure is connected to the understanding and use of a number line or scale. This is different to a number strip in that the number strip can only represent the natural numbers.

1	2	3	4	5	6	7	8

A number line represents whole number and fractions
The number is represented by a unit length and the numberis written at the end of the unit. This allows for a zero and fractions. Students need to come to the understanding that fractions are numbers between the whole numbers.

The words half, quarter and third are not clearly linked to their symbol and therefore naming these early fractions is not obvious to students. It will require plenty of practice to link the words and symbols to the number of equal parts. (Using Te reo Maori names for fractions makes the link much clearer). Fraction words (except half and quarter) are also homonyms being a position in a sequence after first and second. This can add another layer of confusion for students as they are more likely to be familiar with the positional context of the words.
Name fractions like ¾ as “three quarters” and not “three out of four” as this emphasises the use of whole number language. This naming of the fraction only makes sense when using a fraction to represent a proportion.

Unit 4: Beginning Place Value - Unlocking the Number System

Using 10 as a Counting Set	The Importance of a Group of 10	Addition & Subtraction to 20
Using NZ Money	Exploring Measurement Continued from Unit 3	Exploring 2d & 3D Shapes Continued from Unit 3

A multi digit number is the result of adding together the value of the digits in each of the column. It is therefore important that students are able to name each of the columns. To limit the columns knowledge to just tens and ones is likely to mask the difficulty of dealing with zero. Students may continue to think as 10 as ten groups of one and never as one group of ten, likewise 23 is only 23 groups of one. This thinking is masked as students can often say 2 tens because there are 2 tens in the tens column but when given the number 423 they still only see 2 tens in the number and there is no thinking about groups of ten. Anecdotally teachers often say students seem to get place value with two digit numbers but it all falls over with three digit numbers. When working with two digit numbers periodically extend thinking to the three digit to force a change in thinking using materials and to check they are not solely focused on the digit in the tens column but the number of groups of ten in the whole number.
As students develop an understanding of place value they need to be able to view a two digit number in many different ways
For example the number 23
Twenty three ones
Twenty three as 23
23 as 2 tens and 3 ones (where ten is a counting unit)
23 as 20 + 3
23 as 2 groups of ten and 3 groups of one

The decade numbers do not challenge a student’s thinking about the use of zero and the work at this level is only the beginnings of place value. The biggest difficulty for students at this level is maintaining the number in the group, the number of groups and the total. For place value the groups size is a constant – a group of ten and students need to come to the understanding that a groups of ten plays a significant part in our number system and is key to the code of how our numbers are represented.

Earlier work of equal grouping should benefit student thinking in equal sized groups and in this unit the thinking is put towards the understanding of the number system – the multiplicative structures and the additive structures within the system.

Sequencing numbers to 100 was met in Unit Two so students should be able to read and write two digit numbers. Listen carefully for continuing confusion between teen and ty numbers and further investigation and modelling of - ty numbers as groups of ten (multiplicative structure) and - teen numbers as 10 + (additive structure) is likely to be necessary for some students.

To understand 30 as 3 groups of ten is the ten times table, counting multiples of ten. Students need to come to the understanding that the basic facts are repeated in each of the columns.
If students know 3 + 3 = 6, then it follows that 30 + 30 = 60. However the language does not support this thinking, students have to understand the symbolic representation of the decade numbers. Three and three is six does not equate to thirty and thirty is sixty. Hundreds and thousands are easier as saying three hundred and three hundred is six hundred is no different to saying three apples and three apples is six apples.

Students need to understand there are only ten digits 0 – 9 and where they are placed in a number the digit takes on a specific value; the place value of a digit.
0 – 9 are also referred to as numbers – they are single digit numbers. Numbers like 23 are made up of two digits and not two numbers. This gives rise to the term 2 digit number. A zero in a two digit number can only represent no groups of one. The absence of groups of ten is not represented until you have a three digit number.
Without zero we do not have our number system. Understanding the use of zero in a number is fundamental in reading and writing numbers and understanding the additive structure of the Hindu Arabic system. Students will need to have a rudimentary understanding of zero to make sense of expanding numbers into their component parts. 23 = 20 + 3
Students require many opportunities to make sense of the symbols used to represent numbers. Understanding the importance of a group of ten is fundamental. Working with equal groups is a fundamental multiplicative idea. The equal group size is based around ten. (The Arabic Hindu number system is a Base 10 system) Two digit place value is just the first group of tens. The regrouping of the groups of ten into another group of ten gives rise to the third column (hundreds). The system is based on continual regrouping of tens or a nesting of groups of ten within groups of ten.

Students do not readily think of one as a group. But the first column represents groups of one. One is known as the multiplicative identity, when you multiply by one you do not change the number. Using the commutative property of multiplication, 5 x 1 = 1 x 5, Students readily see, and all prior experience has taught them to see one group of five but not the five groups of one.

Real life experiences in the classroom will often reinforce the idea that you cannot have a group of 1 as you either work on your own or in a group.

Students need many early experiences to build a foundational understanding of place value. Continuing conceptual understanding of place value through to generalising the number system with whole numbers and decimals will take many years and good teaching and learning experiences.

The addition and subtraction element extends earlier experiences from Unit One and students need further experiences of all three structures of addition and subtraction with result unknown, change unknown and start unknown type problems. (See introduction to unit one for explanation of the three structures).
Equalising problems are also included in this unit as students begin to use the equals symbol as a relationship between two totals.

Extract from ERO document: Making it Count: Teaching Maths in Year 1 - 3
(https://evidence.ero.govt.nz/documents/making-it-count-teaching-maths-in-years-1-to-3)

Teachers are not as well set up as they could be for the deliberate, structured approach to maths teaching that the evidence tells us makes the difference.

Maths in early primary school matters.Maths achievement in the primary years is linked to later success across a range of life outcomes, like higher education achievement, better jobs, better income, and social mobility. Maths results have even been shown to impact on national economies.

In the early years of primary school, teachers have the opportunity to set the scene for their young maths students through purposeful strategies and explicit instruction. It is in these early years that students learn about the building blocksof all future maths learning, and develop their understanding of how capable they are as maths students. Maths learning builds on itself and gets more complex over a student's time in school, so getting the foundation right is really important. Any misunderstandings, shortcuts, poor self-belief, or lack of engagement in these early years sets a poor foundation for years to come.
To make this happen, teachers need to ensure that all students benefit from high-quality maths experiences every day. When maths is a consistent and engaging feature of the daily classroom programme, students have lots of opportunities to make connections, cement new learning, think and talk in maths terms, and explore maths ideas. Purposeful and evidence-based maths practices are the key to more confident maths students in the future.

There are two enablers for great maths teaching practice

These are the enablers that need to be in place before teachers can do their best maths teaching.

→Enabler 1: Teacher knowledge
Teachers need to be confident in their own maths knowledge and skills, to be ready to teach them to students. They also need to understand what works best for young students: the specific teaching strategies that are most effective in setting students up for this crucial time in their maths journey. This includes being clear about how to structure their teaching to develop important maths understandings over time, while avoiding misunderstandings or shortcuts which negatively impact on later learning.

→Enabler 2: School culture and a whole school curriculum
Teachers' school settings can promote good maths practice through a clear, shared understanding of quality maths teaching. This involves clearly setting out what maths teaching and maths progress looks like in a documented, structured whole school curriculum, and by supporting teacher understanding with great learning and collaboration opportunities. It's useful when schools have an embedded culture of being open to learning, sharing, and continually improving.

